Cours Electromagnétisme 2, séance 1

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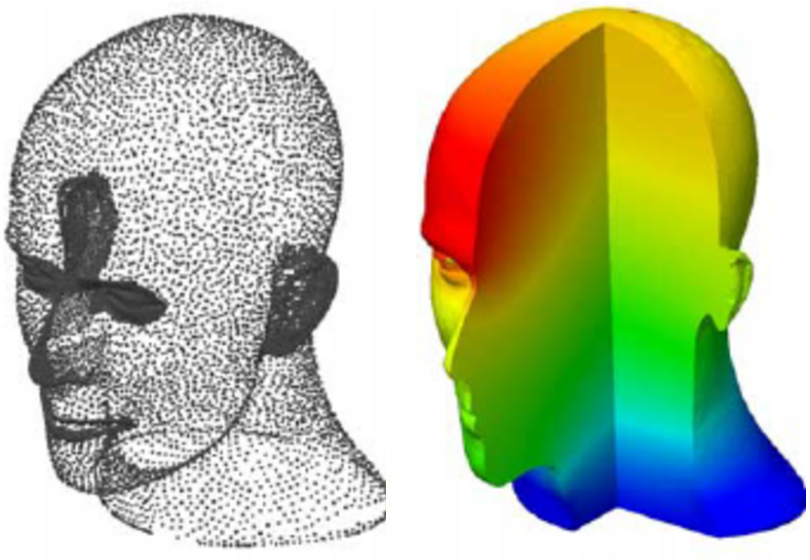
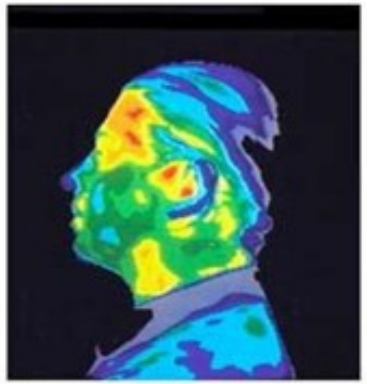
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

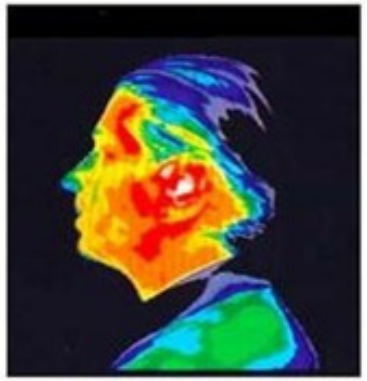
LWE Laboratory of Wave Engineering

Champs scalaires



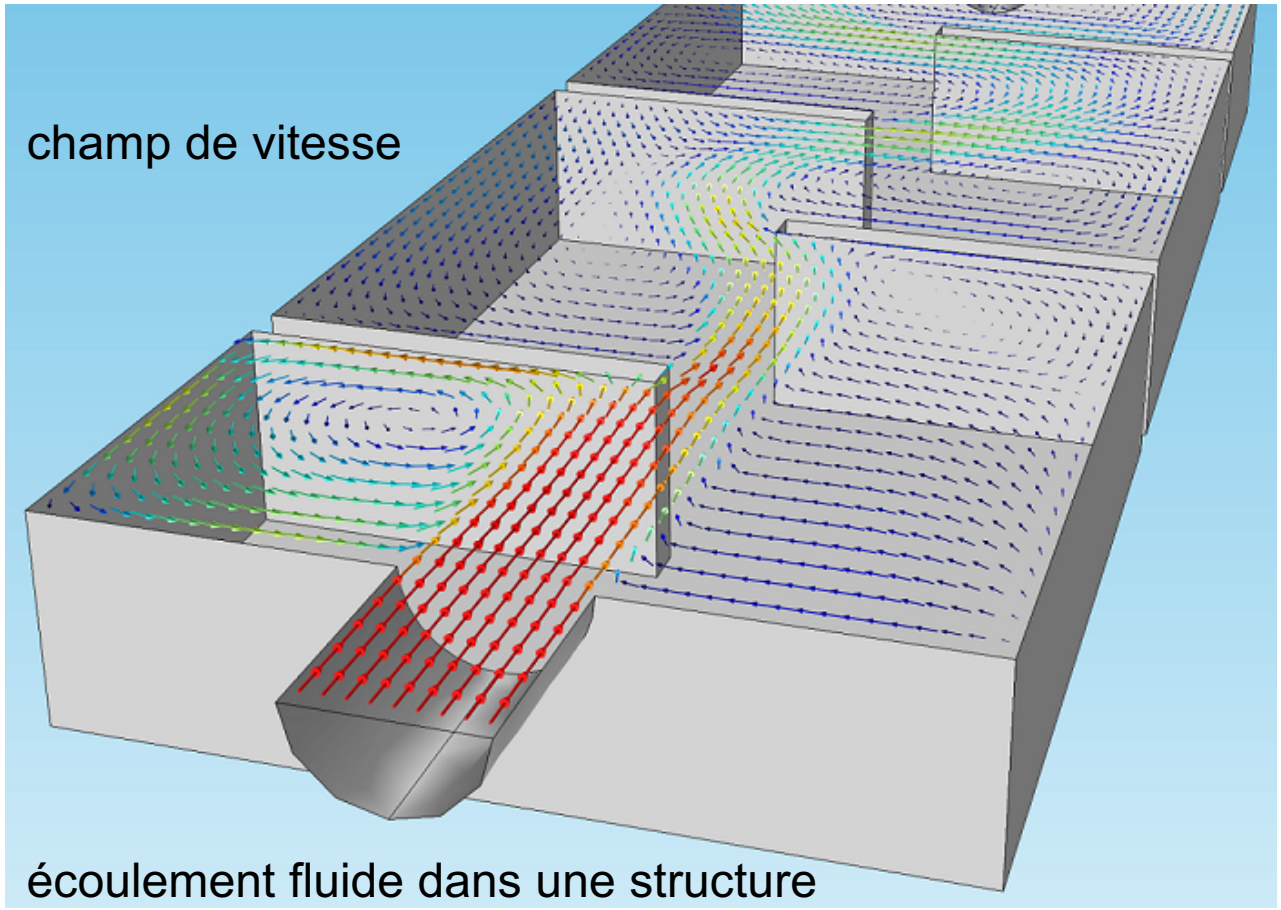


Thermographic Image of the head with no exposure to harmful cell phone radiation.

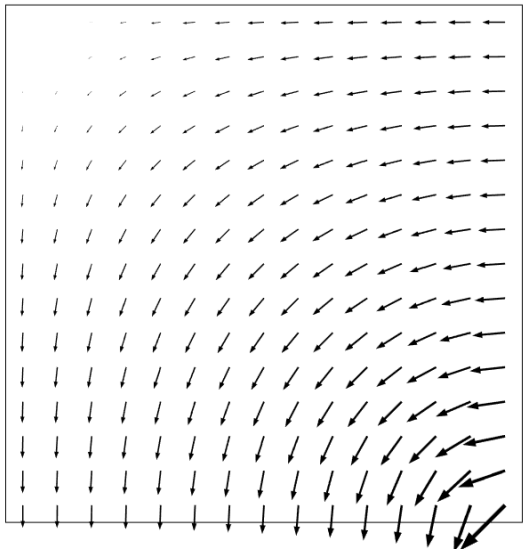


Thermographic Image of the head after a 15-minute phone call. Yellow and red areas indicate thermal (heating) effects that can cause negative health effects.

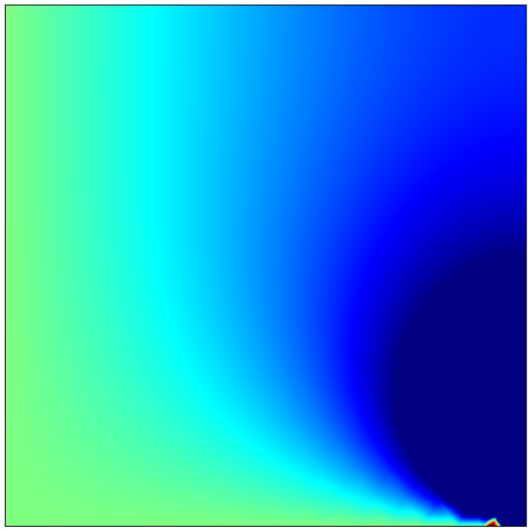
Champs vectoriels



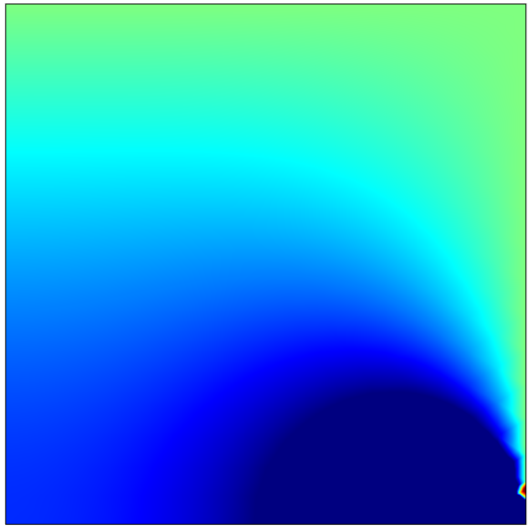
Champs vectoriels



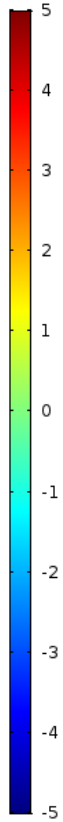
champ vectoriel



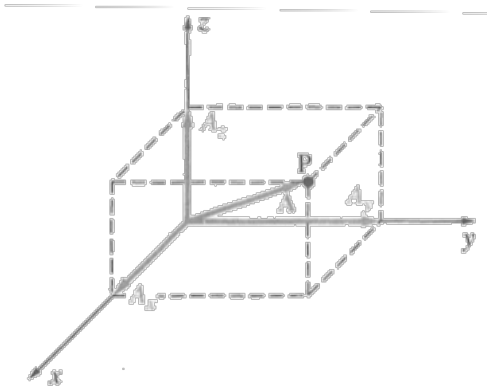
composante x
=champ scalaire



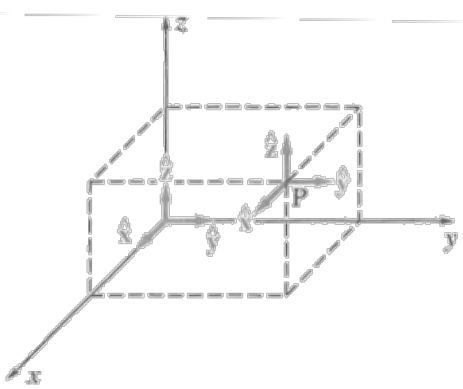
composante y
=champ scalaire



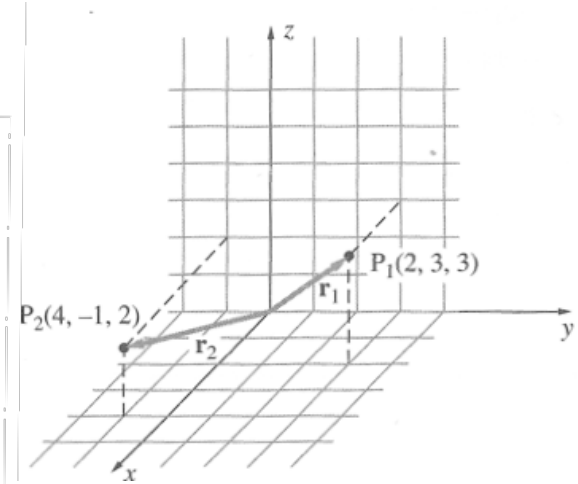
Système cartésien



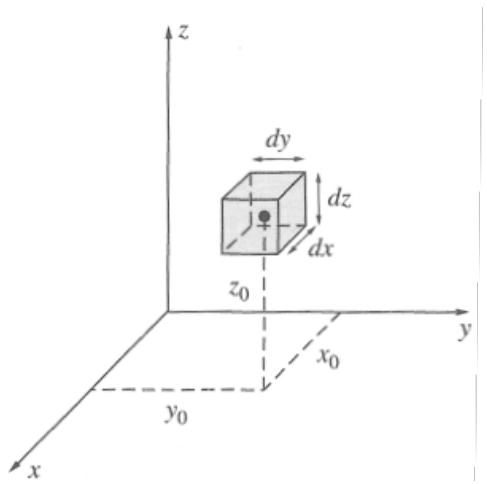
repérage de vecteurs



vecteurs de base



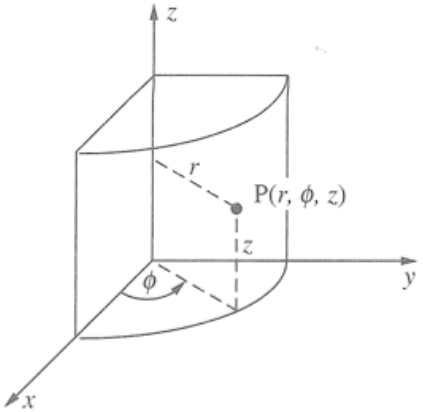
repérage de points



volume élémentaire

Système cylindrique

coordonnées



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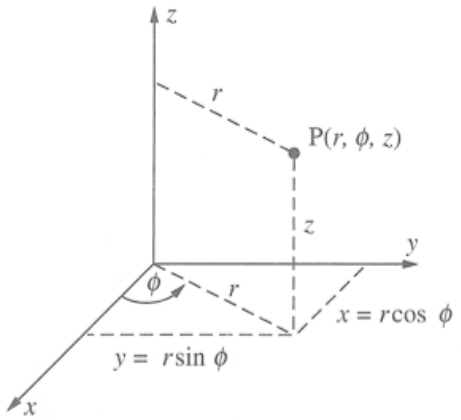
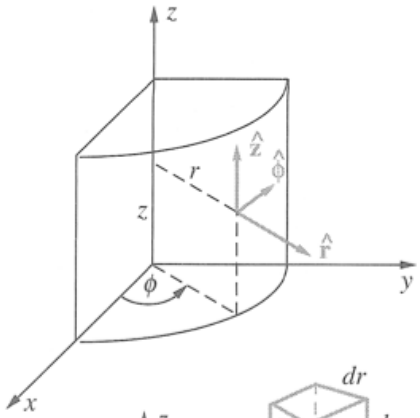
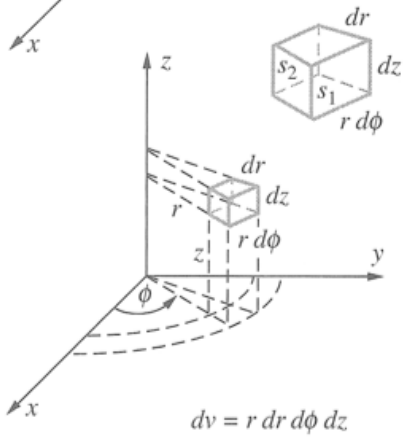


vecteurs de base



volume élémentaire



Système cylindrique – formules utiles

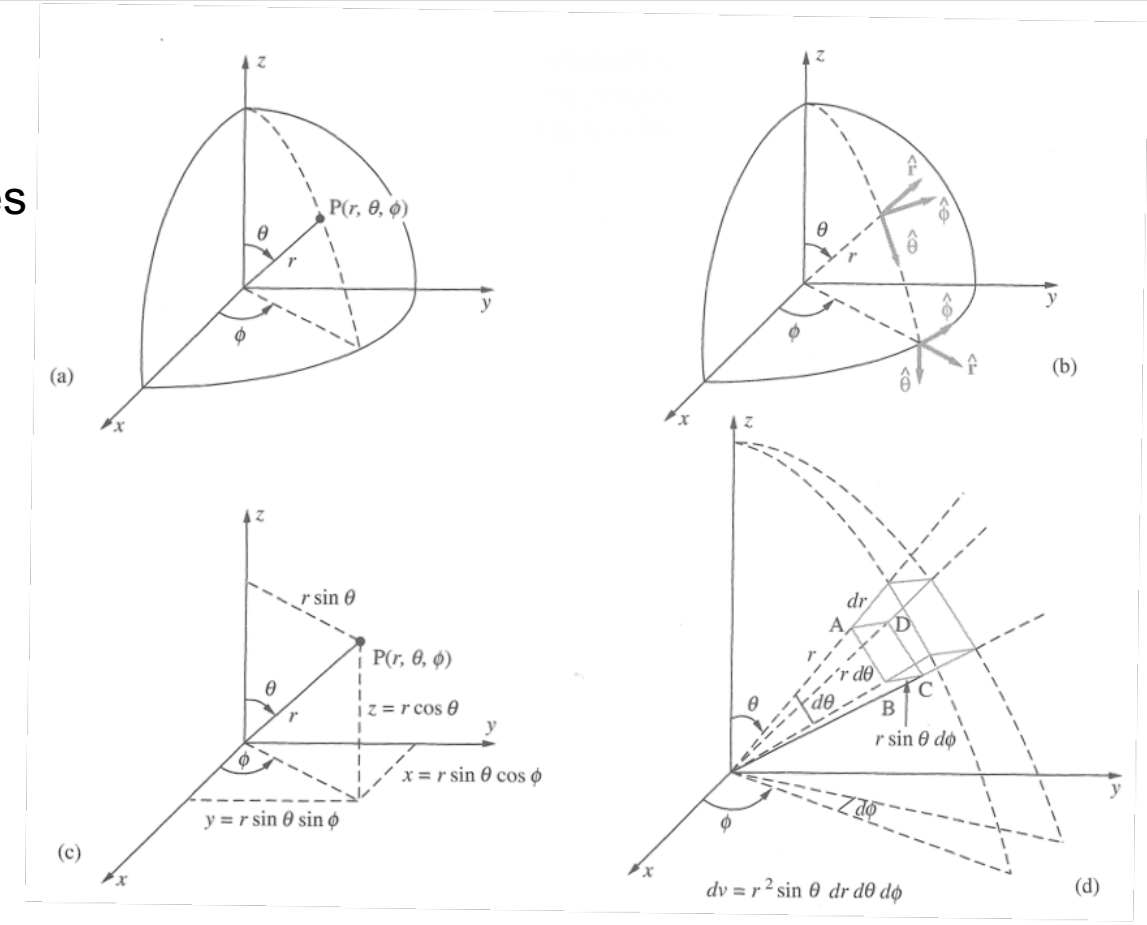
Type de transformation	Cartésiennes à cylindriques	Cylindriques à cartésiennes
Coordonnées	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$
Vecteurs unitaires	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}.$

Volume élémentaire: $dV(P) = r dr\, d\phi\, dz$

Système sphérique

coordonnées

repérage de points



vecteurs de base

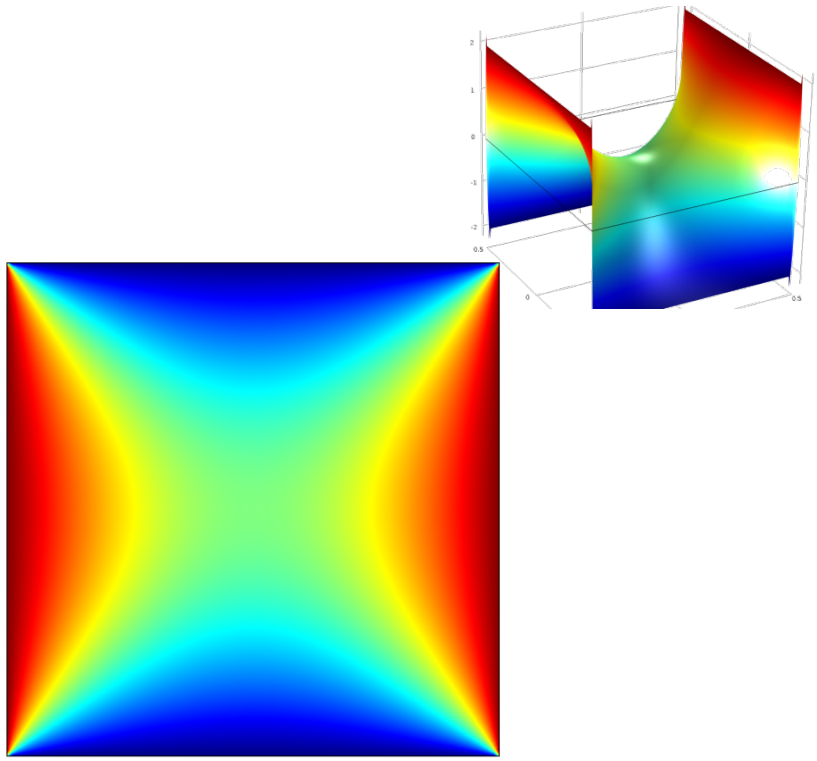
volume élémentaire

Système sphérique – formules utiles

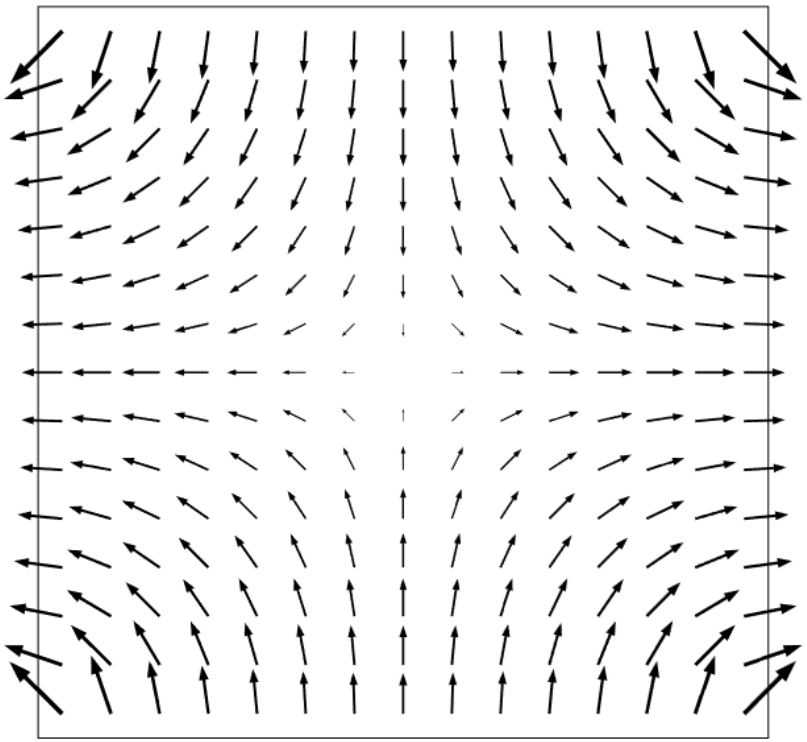
Type de transformation	Cartésiennes à sphériques	Sphériques à cartésiennes
Coordonnées	$r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \cos^{-1}(z/\sqrt{x^2 + y^2 + z^2})$ $\phi = \tan^{-1}(y/x)$	$x = r\sin\theta\cos\phi\,,$ $y = r\sin\theta\sin\phi\,,$ $z = r\cos\theta$
Vecteurs unitaires	$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$\hat{x} = \hat{r}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta\,.$

Volume élémentaire: $dV(P) = r^2\sin\theta\,dr\;\;d\theta\,d\phi.$

Gradient

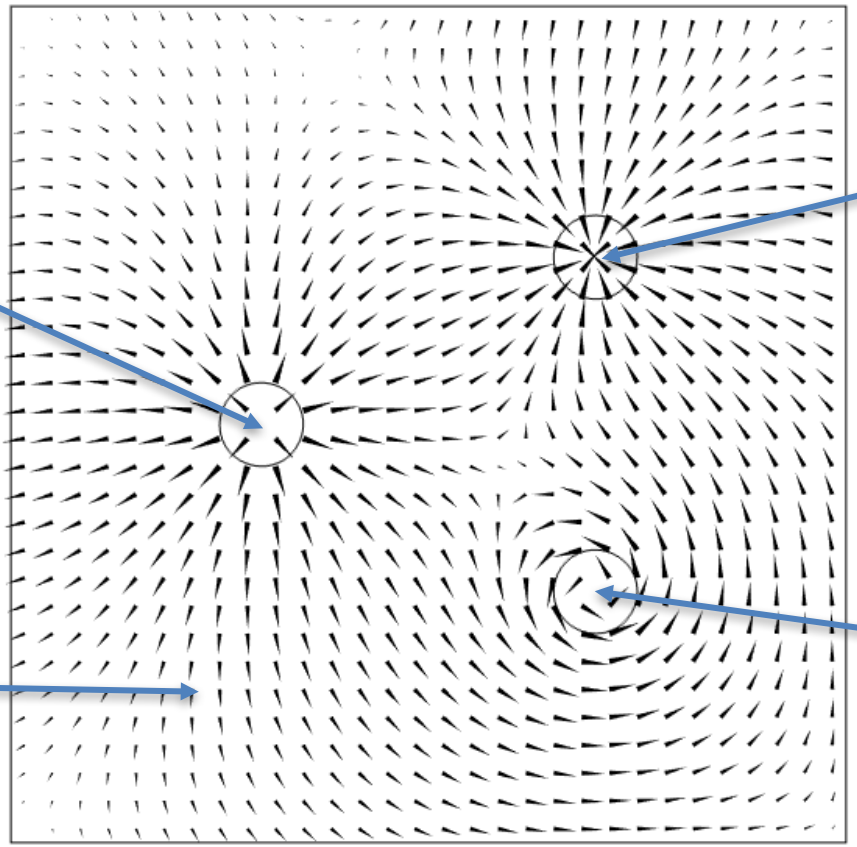


champ scalaire



gradient
=champ vectoriel

Divergence et rotationnel



Formules utiles

	Gradient $\vec{\nabla}T$	Divergence $\vec{\nabla}.\vec{v}$	Rotationnel $\vec{\nabla}\times\vec{v}$
Cartésien	$\frac{\partial T}{\partial x}\hat{x}+\frac{\partial T}{\partial y}\hat{y}+\frac{\partial T}{\partial z}\hat{z}$	$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}$	$\left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\hat{x}$ $+\left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\hat{y}$ $+\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\hat{z}$
Cylindrique	$\frac{\partial T}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi}+\frac{\partial T}{\partial z}\hat{z}$	$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\phi}{\partial \phi}+\frac{\partial v_z}{\partial z}$	$\left(\frac{1}{r}\frac{\partial v_z}{\partial \phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{r}$ $+\left(\frac{\partial v_r}{\partial z}-\frac{\partial v_z}{\partial r}\right)\hat{\phi}$ $+\frac{1}{r}\left(\frac{\partial}{\partial r}(rv_\phi)-\frac{\partial v_r}{\partial \phi}\right)\hat{z}.$
Sphérique	$\frac{\partial T}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta}$ $+\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta)+$ $\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(v_\phi\sin\theta)-\frac{\partial v_\theta}{\partial \phi}\right)\hat{r}$ $+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{\partial}{\partial r}(rv_\phi)\right)\hat{\theta}$ $+\frac{1}{r}\left(\frac{\partial}{\partial r}(rv_\theta)-\frac{\partial v_r}{\partial \theta}\right)\hat{\phi}.$